Таблица производных основных элементарных функций

k, α, a – вещественные константы, $a > 0$, $a \neq 1$;
$u = u(x)$ – функция, аргумент сложной функции.

Производные основных элементарных функций	Производные сложных функций
$k' = 0$; $x' = 1$;	
$(x^\alpha)' = \alpha x^{\alpha-1}$;	$(u^\alpha)' = \alpha u^{\alpha-1} \cdot u'$;
$(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}$;	$(\sqrt{u})' = \dfrac{1}{2\sqrt{u}} \cdot u'$;
$(a^x)' = a^x \ln a$;	$(a^u)' = a^u \ln a \cdot u'$;
$(e^x)' = e^x$;	$(e^u)' = e^u \cdot u'$;
$(\log_a x)' = \dfrac{1}{x \ln a}$;	$(\log_a u)' = \dfrac{1}{u \ln a} \cdot u'$;
$(\ln x)' = \dfrac{1}{x}$;	$(\ln u)' = \dfrac{1}{u} \cdot u'$;
$(\sin x)' = \cos x$;	$(\sin u)' = \cos u \cdot u'$;
$(\cos x)' = -\sin x$;	$(\cos u)' = -\sin u \cdot u'$;
$(\operatorname{tg} x)' = \dfrac{1}{\cos^2 x}$;	$(\operatorname{tg} u)' = \dfrac{1}{\cos^2 u} \cdot u'$;
$(\operatorname{ctg} x)' = -\dfrac{1}{\sin^2 x}$;	$(\operatorname{ctg} u)' = -\dfrac{1}{\sin^2 u} \cdot u'$
$(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$;	$(\arcsin u)' = \dfrac{1}{\sqrt{1-u^2}} \cdot u'$;
$(\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}}$;	$(\arccos u)' = -\dfrac{1}{\sqrt{1-u^2}} \cdot u'$;
$(\operatorname{arctg} x)' = \dfrac{1}{1+x^2}$;	$(\operatorname{arctg} u)' = \dfrac{1}{1+u^2} \cdot u'$;
$(\operatorname{arcctg} x)' = -\dfrac{1}{1+x^2}$;	$(\operatorname{arcctg} u)' = -\dfrac{1}{1+u^2} \cdot u'$.

Примеры нахождения производных элементарных функций

$\boxed{(k)' = 0}$ $\quad (3)' = 0$; $\quad (8)' = 0$;

$\boxed{(x)' = 1}$ $\boxed{(kx)' = k}$ $\quad (3x)' = 3$; $\quad (8x)' = 8$;

$\boxed{(x^a)' = a \cdot x^{a-1}}$ $\quad (x^2)' = 2x$; $\quad (x^3)' = 3x^2$ $\quad (x^4)' = 4x^3$;

$(5x^2)' = 5 \cdot 2x = 10x$; $\quad (5x^3)' = 5 \cdot 3x^2 = 15x^2$;

$(\sqrt{x})' = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$; $\quad (\sqrt[5]{x^3})' = \left(x^{\frac{3}{5}}\right)' = \frac{3}{5} x^{-\frac{2}{5}} = \frac{3}{5\sqrt[5]{x^2}}$;

$\left(\frac{1}{x}\right)' = (x^{-1})' = (-1) \cdot x^{-2} = -\frac{1}{x^2}$;

$\left(\frac{1}{\sqrt{x}}\right)' = \left(x^{-\frac{1}{2}}\right)' = -\frac{1}{2} x^{-\frac{3}{2}} = -\frac{1}{2\sqrt{x^3}}$;

$\boxed{(\ln x)' = \frac{1}{x} = x^{-1}}$ $\quad (5 \ln x)' = 5 \cdot \frac{1}{x}$; $\quad (3 \ln x)' = \frac{3}{x}$;

$\boxed{(e^x)' = e^x}$ $\quad (7e^x)' = 7e^x$; $\quad (8e^x)' = 8e^x$;

$\boxed{(a^x)' = a^x \ln a}$ $\quad (2^x)' = 2^x \ln 2$; $\quad (6^x)' = 6^x \ln 6$;

$\boxed{(\sin x)' = \cos x}$ $\quad (4 \sin x)' = 4 \cos x$; $\quad (8 \sin x)' = 8 \cos x$;

$\boxed{(\cos x)' = -\sin x}$ $\quad (4 \cos x)' = -4 \sin x$; $\quad (8 \cos x)' = -8 \sin x$;

$\boxed{(\operatorname{tg} x)' = \frac{1}{\cos^2 x}}$ $\quad \boxed{(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}}$ $\quad \boxed{(\operatorname{arctg} x)' = \frac{1}{1+x^2}}$

$\boxed{(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}}$ $\quad \boxed{(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}}$ $\quad \boxed{(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}}$

Примеры нахождения производной произведения и частного

$$\boxed{(u\cdot v)' = u'v + uv'}$$

$$\left(x^5\cdot \sin x\right)' = \left(x^5\right)'\cdot \sin x + x^5\cdot(\sin x)' = 5x^4\cdot \sin x + x^5\cdot \cos x \ ;$$

$$\left((2x+7)(3x+5)\right)' = (2x+7)'(3x+5) + (2x+7)(3x+5)' = 2\cdot(3x+5) + (2x+7)\cdot 3 \ ;$$

$$\boxed{\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}}$$

$$\left(\frac{x^5}{\sin x}\right)' = \frac{\left(x^5\right)'\cdot \sin x - x^5\cdot(\sin x)'}{\sin^2 x} = \frac{5x^4\cdot \sin x - x^5\cdot \cos x}{\sin^2 x} \ ;$$

$$\left(\frac{3x+4}{5x+2}\right)' = \frac{(3x+4)'(5x+2) - (3x+4)(5x+2)'}{(5x+2)^2} = \frac{3\cdot(5x+2) - (3x+4)\cdot 5}{(5x+2)^2} \ ;$$

Примеры нахождения производной сложной функции

$$\boxed{(\sin u)' = \cos u\cdot u'} \qquad (\sin 3x)' = \cos 3x\cdot(3x)' = \cos 3x\cdot 3 \ ;$$

$$(\sin(3x-2))' = \cos(3x-2)\cdot(3x-2)' = \cos(3x-2)\cdot 3 \ ;$$

$$(\sin(\ln x))' = \cos(\ln x)\cdot(\ln x)' = \cos(\ln x)\cdot\frac{1}{x} \ ;$$

$$\left(\sin(x^3+6x-2)\right)' = \cos(x^3+6x-2)\cdot(3x^2+6) \ ;$$

$$\boxed{(\cos u)' = -\sin u\cdot u'} \qquad (\cos 2x)' = -\sin 2x\cdot(2x)' = -\sin 2x\cdot 2 \ ;$$

$$(\cos(2x+3))' = -\sin(2x+3)\cdot(2x+3)' = -\sin(2x+3)\cdot 2 \ ;$$

$$(\cos(\sqrt{x}))' = -\sin(\sqrt{x})\cdot(\sqrt{x})' = -\sin(\sqrt{x})\cdot\frac{1}{2\sqrt{x}} \ ;$$

$$\left(\cos(x^2+3x-5)\right)' = -\sin(x^2+3x-5)\cdot(2x+3) \ ;$$

$$\boxed{\left(e^u\right)' = e^u\cdot u'} \qquad \left(e^{5x}\right)' = e^{5x}\cdot(5x)' = e^{5x}\cdot 5 \ ;$$

$$\left(e^{5x-7}\right)' = e^{5x-7}\cdot(5x-7)' = e^{5x-7}\cdot 5 \ ;$$

$$\left(e^{x^2+4x+1}\right)' = e^{x^2+4x+1}\cdot(2x+4) \ ;$$

$$\left(e^{\sin x}\right)' = e^{\sin x}\cdot \cos x \ ;$$

$$\boxed{(\ln u)' = \frac{1}{u}\cdot u'} \qquad (\ln 2x)' = \frac{1}{2x}\cdot 2 = \frac{1}{x}; \qquad (\ln 7x)' = \frac{1}{7x}\cdot 7 = \frac{1}{x};$$

$$(\ln(3x+4))' = \frac{1}{3x+4}\cdot(3x+4)' = \frac{1}{3x+4}\cdot 3 = \frac{3}{3x+4} \ ;$$

$$\left(\ln(x^2+6x-3)\right)' = \frac{1}{x^2+6x-3}\cdot(2x+6) = \frac{2x+6}{x^2+6x-3} \ ;$$

$$(\ln(\cos x))' = \frac{1}{\cos x}\cdot \sin x = \frac{\sin x}{\cos x} = \operatorname{tg} x \ .$$