**Таблица неопределенных интегралов**

$k, \alpha, a$ – вещественные константы, $a > 0$, $a \neq 1$; $\alpha \neq -1$.

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|---|---|---|---|
| **1.** | $\int du = u + C$<br>$\int k du = ku + C$ | **2.** | $\int 0\, du = C$ |
| **3.** | $\int u^{\alpha} du = \frac{u^{\alpha+1}}{\alpha+1} + C,\ \alpha \neq -1$<br>$\int u^{-1} du = \int \frac{1}{u} du = \ln \lvert u \rvert + C$ | **4.** | $\int \frac{du}{u^2} = -\frac{1}{u} + C,$<br>$\int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C$ |
| **5.** | $\int a^u du = \frac{a^u}{\ln a} + C$ | **6.** | $\int \mathrm{e}^u du = \mathrm{e}^u + C$ |
| **7.** | $\int \sin u\, du = -\cos u + C$ | **8.** | $\int \cos u\, du = \sin u + C$ |
| **9.** | $\int \frac{du}{\cos^2 u} = \operatorname{tg} u + C$ | **10** | $\int \frac{du}{\sin^2 u} = -\operatorname{ctg} u + C$ |
| **11.** | $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \operatorname{arctg} \frac{u}{a} + C$ | **12.** | $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left\lvert \frac{u+a}{u-a} \right\rvert + C$ |
| **13.** | $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{u-a}{u+a} \right\rvert + C$ | **14.** | $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$ |
| **15.** | $\int \frac{du}{\sqrt{u^2 + a^2}} = \ln \left\lvert u + \sqrt{u^2 + a^2} \right\rvert + C$ | **16.** | $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln \left\lvert u + \sqrt{u^2 - a^2} \right\rvert + C$ |
| **17.** | $\int \frac{du}{\sin u} = \ln \left\lvert \operatorname{tg} \frac{u}{2} \right\rvert + C$ | **18.** | $\int \frac{du}{\cos u} = \ln \left\lvert \operatorname{tg} \left( \frac{u}{2} + \frac{\pi}{4} \right) \right\rvert + C$ |
| **19.** | $\int \operatorname{tg} u\, du = -\ln \lvert \cos u \rvert + C$ | **20.** | $\int \operatorname{ctg} u\, du = \ln \lvert \sin u \rvert + C$ |

***Примеры нахождения интегралов***

$\boxed{\int 0 dx = C}$

$\boxed{\int dx = x + C}$ $\qquad$ $\boxed{\int k dx = kx + C}$

$\int 3dx = 3x + C$ ’ $\qquad$ $\int 8dx = 8x + C$ ;

$\boxed{\int x^a dx = \frac{x^{a+1}}{a+1} + C}$

$\int x dx = \frac{x^2}{2} + C$ ;

$\int x^2 dx = \frac{x^3}{3} + C$ ;

$\int x^3 dx = \frac{x^4}{4} + C$ ;

$\int x^4 dx = \frac{x^5}{5} + C$ ;

$\int x^5 dx = \frac{x^6}{6} + C$ ;

$\int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{3/2} + C = \frac{2}{3} x\sqrt{x} + C$ ;

$\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$ ;

$\int 3x dx = 3\frac{x^2}{2} + C$ ;

$\int 5x^3 dx = 5\frac{x^4}{4} + C$ ;

$$\int \frac{dx}{x} = \ln|x| + C \qquad \int \frac{1}{x}dx = \ln|x| + C \qquad \int x^{-1}dx = \ln|x| + C$$

$$\int \frac{3dx}{x} = 3\ln|x| + C; \qquad \int \frac{5}{x}dx = 5\ln|x| + C;$$

$$\int e^x dx = e^x + C \qquad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int 3^x dx = \frac{3^x}{\ln 3} + C; \qquad \int 5^x dx = \frac{5^x}{\ln 5} + C;$$

$$\int \sin x dx = -\cos x + C \qquad \int 5\sin x dx = -5\cos x + C;$$

$$\int \cos x dx = \sin x + C \qquad \int 4\cos x dx = 4\sin x + C;$$

$$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C \qquad \int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$$

**«Инженерные» интегралы**

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \qquad \int \frac{dx}{x^2 + a^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C$$

$$\int \frac{dx}{9 + x^2} = \frac{1}{3}\operatorname{arctg}\frac{x}{3} + C; \qquad \int \frac{dx}{x^2 + 16} = \frac{1}{4}\operatorname{arctg}\frac{x}{4} + C;$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C \qquad \int \frac{dx}{x^2 - 9} = \frac{1}{6}\ln\left|\frac{x-3}{x+3}\right| + C;$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C \qquad \int \frac{dx}{\sqrt{9 - x^2}} = \arcsin\frac{x}{3} + C;$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left|x + \sqrt{x^2 - a^2}\right| + C \qquad \int \frac{dx}{\sqrt{x^2 - 25}} = \ln\left|x + \sqrt{x^2 - 25}\right| + C;$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left|x + \sqrt{x^2 + a^2}\right| + C \qquad \int \frac{dx}{\sqrt{x^2 + 16}} = \ln\left|x + \sqrt{x^2 + 16}\right| + C;$$

**Сопоставление формул**

$$\int k \cdot f(x)dx = k \cdot F(x) + C \quad \textbf{и} \quad \int f(kx + b)dx = \frac{1}{k}F(kx + b) + C$$

$$\int e^{kx+b}dx = \frac{1}{k}e^{kx+b} + C \qquad \int e^{5x}dx = \frac{1}{5}e^{5x} + C;$$

$$\int e^{5x-7}dx = \frac{1}{5}e^{5x-7} + C, \quad \text{но} \quad \int 5e^x dx = 5e^x + C;$$

$$\int \cos(kx + b)dx = \frac{1}{k}\sin(kx + b) + C \qquad \int \cos 2x dx = \frac{1}{2}\sin 2x + C;$$

$$\int \cos(2x + 3)dx = \frac{1}{2}\sin(2x + 3) + C, \quad \text{но} \quad \int 2\cos x dx = 2\sin x + C;$$

$$\int \sin(kx + b)dx = -\frac{1}{k}\cos(kx + b) + C \qquad \int \sin 3x dx = -\frac{1}{3}\cos 3x + C;$$

$$\int \sin(3x - 2)dx = -\frac{1}{3}\cos(3x - 2) + C, \quad \text{но} \quad \int 3\sin x dx = -3\cos x + C.$$